ON THE EARLY HISTORY OF THE DECIMAL POINT

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The early history of the decimal fraction has been considered so extensively that it would seem that little could remain unsaid concerning its inventors. Even in the less important matter of symbolism, the ground would seem to have been covered, thanks to the investigations of men like Braunmühl, Cajori, Eneström, Glaisher, Karpinski, Smith, and Tropfke. As to the notion of the fraction itself, the facsimiles in Smith’s History of Mathematics (Vol. 2 pp. 236–243, with the accompanying text) set forth the case with considerable completeness, showing that such fractions were used some time before the decimal point was suggested. The latter is merely one of several symbols used to separate the fractional and integral parts, and the question as to what device shall be selected is merely one relating to convenience.

The purpose of this article is to call attention to a fact which seems to have been overlooked by all historians, namely, that the credit of being the first to use a dot (point) as a separatrix, with a full knowledge of its significance in this connection, is due to the Jesuit father Christopher Clavius (1537–1612). In his work on the astrolabe, published in Rome in 1593 he gives (pp. 195–270) a “Tabula Sinuum,” where the proportional parts are separated from the integers by periods (points, dots). A portion of the table is here shown. So far as now known, this is the first appearance of the decimal point in a work in which its full significance is given. The table antedates the Pitiscus edition of 1608, which has been so carefully studied by Professor Cajori.¹ It also antedates Napier’s Rhabdologia (1617), Wright’s translation of the Mirifici logarithmorum canonis descriptio (1616), and Kepler’s Ausszug aus der urwählen Messe-Kunst Archimedis (1616), in each of which the decimal point or comma is used. As to Bürgi’s use of the symbol in a manuscript of 1592, the claim is no longer seriously considered.² The name of Stevin, who wrote the first book upon decimal fractions,² need not be considered in this connection since he made no use of the symbolism under discussion.

It remains to consider whether Clavius understood the significance of the symbol, for otherwise he would be entitled to but little more credit than Pellos who, in 1492, wrote 538694.3 as the quotient of 53836943 by 10, but showed no further appreciation of decimals. That Clavius actually did understand its significance appears by the following statement on page 228:

---quoniam inter duos sinus grad. 16 min. 12 grad. 16 min. 13 positi sunt duo binumeri 46.5 colligemus uni secundo inter minutum 12 13 gradus 16 congrue particulas 46.5/10, ex differentio 2793 inter duos sinus

³ Le Thiende, Antwerp, 1535, with a French translation (La disme) in the same year.
Facsimile of p. 198 of the “Astrolabe” of Clavius (1593)
It amounts to the following:

In the table on p. 198, the number 46.5 is put between \(\sin 16^\circ 12'\) and \(16^\circ 13'\). Hence to every second (of the difference) between \(16^\circ 12'\) and \(16^\circ 13'\) we will apportion \(46\,\%\) particles of the difference (2793) between 2789911 (\(\sin 16^\circ 12'\)) and 2792704 (\(\sin 16^\circ 13'\)) etc.

It would seem, therefore, that up to the present time the evidence is conclusive that Clavius was the first to use the decimal point with a clear understanding of its significance. This statement does not, of course, invalidate the positive evidence that the decimal fraction was used with other symbols long before his time.

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**PROPERTIES OF TWO POINTS ASSOCIATED WITH A TRIANGLE**

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The Brocard points of a triangle are defined as the intersections of two triads of circles described on \(BC, CA, AB\) as chords in such a way that the external segments of the circles contain the angles (1) \(C, A, B\), and (2) \(B, C, A\) respectively.\(^1\)

If the triads of circles are drawn in such a way that the internal segments contain the angles (1) \(C, A, B\) and (2) \(B, C, A\), respectively, two other points \(P_2\) and \(P_4\) will be determined. It is the purpose of the present paper to discuss some of the properties of the points \(P_2\) and \(P_4\).

**Theorem I:** The pedal triangles of \(P_2\) and \(P_4\) are similar.

**Proof:** Since in the figure \(P_2\) lies within the angle \(BCA\) and \(D_2E_2F_2\) is the pedal triangle of \(P_2\) we have \(\angle BPC = \angle (A + D_2)\). Therefore \(\angle D_2 = \angle (C - A)\).

Similarly \(\angle E_2 = \angle (A - B); \quad \angle F_2 = \angle (\pi + B - C)\). Also if, \(D_4E_4F_4\) is the pedal triangle of \(P_4\) it may be shown in the same way that

\[
\angle D_4 = \angle (A - B), \quad \angle E_4 = \angle (\pi + B - C), \quad \angle F_4 = \angle (C - A).
\]

Hence the pedal triangles of \(P_2\) and \(P_4\) are similar.

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