CONCERNING THE BIBLIOGRAPHY OF MATHEMATICS. *

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I. THE CHARACTER OF THE BIBLIOGRAPHIC PROBLEM IN MATHEMATICS.

Mathematics is without doubt the most impersonal of all the branches of human thought. "Views" are not tolerated within its domain; "schools of thought" in the ordinary acceptance of the term are unknown. Conjectures, beliefs, moral certitudes, even, are not given a place among the truths of mathematics, though they are usually more or less distinct stages in the discovery of such truths. Only those statements are admitted to the rank of established mathematical truths, which are accepted by every normal mind to which they are properly presented.

Mathematics has been characterized as the only science which is not concerned about the truth or falsity of its data. The form of reasoning is: If these premises are true, and if no other premises are taken into account, then such a conclusion follows. The second if is very important and is a part of every mathematical inference.

A consequence of the severe standard thus set is that mathematics revises its results little. The statements of Euclid are as valid today as two thousand years ago. The mathematician does not overturn the results of his predecessors; he extends them. He may find that the earlier work did not cover all the possibilities that have been seen later, but he does not on that account impugn the validity of what was done, so far as it went.

A good example is found in Euclid’s postulate of parallels, viz., "Given a point and a straight line, there is one (and only one) straight line passing through the point and parallel to the given straight line."† For many centuries this postulate was accepted as a logical necessity in Geometry, but during the 19th century‡ it was clearly seen that there was no reason why other assumptions should not be made, either that no parallel can be drawn through the given point, or that more than one can be drawn and a consistent Geometry developed. Thus arose the non-Euclidean Geometries. But the validity of the Euclidean Geometry was not at all called into question. The change made was that what was once the geometry became a geometry among several.

I have cited this as an illustration of the general statement: The growth of mathematics has been an evolution without a revolution. Mathematics has as yet had no Copernicus, and expects none.

All this has had a profound influence on mathematical literature. What is written is either a first presentation of a new result, or a representation of it

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* An address delivered before the Chicago Bibliographical Society, April 30, 1903.
† By parallel straight lines, we understand here, straight un terminated lines in the same plane, which do not intersect.
‡ Simon asserts (Jahresbericht d. deutschen Math. Ver. 1896, page 30) that the earliest established recognition that the parallel axiom is not a logical necessity was made by Gauss about 1792, who influenced the Bolyai and Lobatschewsky.
for one of several purposes that need not be enumerated here. The great geniuses of mathematics have not headed opposed and contending schools, but in harmonious, supplementary activity have added new truths to the mathematical treasury. In mathematics, therefore, questions of Bibliography center not about men but about topics. We need not expect to find thousands of works of comment and controversy written in a few years about the theories of any mathematician, however great.

II. What Are the Mathematician’s Bibliographic Needs?

It is evident from what has just been said that the mathematical investigator needs primarily topical bibliographies. To inform ourselves thoroughly as to what has already been done is an almost indispensable preliminary to any serious research. This is needed both to enable the investigator to avail himself of what has already been accomplished, and also to prevent duplication. In other branches the independent re-doing of nominally the same piece of work is likely to be accomplished in such different form and spirit that the value of the one work is diminished little by the existence of the other. In mathematics real duplication is quite possible and, in fact, has frequently occurred.

III. How the Need Is Met.

1. The first refuge is always the *Fortschritte der Mathematik*, issued annually, giving brief descriptions of all available mathematical publications of the year in question. (I shall refer to this journal hereafter simply as the *Fortschritte*). It covers the period back to 1870, and references to all the earlier publications of importance are to be expected in various connections in the publications of that period. These references often amount to a very fair bibliography of the topic to date. I shall mention other publications (made and projected), covering the literature since 1800, which is by far larger and more important part of the whole.

The chief disadvantage of the *Fortschritte* is that it is about three years behind time. (The volume for 1900, for example, has just been completed.) Several other publication bring us nearer to date.

2. The most important of these is the *International Catalogue of Scientific Literature*, published for the International Council by the Royal Society of London. The first annual issue (for 1901) was published November, 1902. It gives titles only, classified on a decimal system, four digits in each number. The issue contains two catalogues, subject and author, each given in four languages, English, French, German, Italian.*

3. The International Congress for Bibliography of Mathematical Science held in Paris, 1889, appointed a commission to secure a list of titles of works from 1800 to 1889, with decennial supplement thereafter. The work done has taken the form of card titles,—published in series of about 1000 each,—and the publication is still in progress.

4. *The Revue Semestrielle des Publications Mathématiques*, published under the

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*A fuller description of this important catalogue will follow in a later number of the Monthly.*
auspices of Mathematical Society of Amsterdam, now in its twelfth volume, comes nearest to covering the literature to date. The part for April to October, 1892, for example, appeared in February, 1903. The journals only are covered, and a few words of abstract are sometimes given.

5. _The Bolletino di Bibliografia e storia delle scienze matematiche_ (in its 6th volume) is given mainly to reviews of new books.

These are the chief periodicals given up to bibliographic work exclusively. Among them, the _Fortschrifte_, the oldest, still remains the most valuable aid so far as it goes, on account of its abstracts which range from a few lines to several pages according to circumstances.

A comparison of numbers of titles may be of some interest. The Royal Society Catalogue for 1901 contains 1506 titles in pure mathematics (2096 in Astronomy). The initial volume is necessarily somewhat incomplete, as quite a complex organization of national bureaus, readers, etc., has to be got into running order. The _Fortschritte_ for 1900 contains (estimate) about 2600 titles, including applied mathematics, geodesy, and astronomy.

6. A most serviceable aid to the working mathematician, especially on the bibliographic side, is _L'Intermédiaire des Mathématiciens_ (monthly; founded in 1894). This journal is intended as a medium for interchange of information between working mathematicians. The form is that of questions and answers. The questions represent genuine needs of the writer arising in his researches; often they are directly bibliographical, while others relate to solution of specific problems. The answers are bibliographical when possible; that is, references which will put the questioner on the track of what he wants; though if a reader can give the derived solution himself, but knows of no previous publication of it, the solution itself will be published as reply if brief; if extended, it is published elsewhere, and reference made in _Intermédiaire_. The _Intermédiaire_ is now completing its first decade; it has met a cordial reception among mathematicians and has done good service in making generally available many pieces of information which would otherwise have remained unutilized.

7. Another phase of general mathematical bibliography is that of classified collections of results, with references to sources for proofs. Many such have been made of varying character and thoroughness, dealing with only a small field; others attempting to cover the entire range of mathematics. I can not now enter upon an enumeration of them; to prepare an exhaustive list would be a bibliographic task in itself. But I mention simply two: Carr, _Synopsis of Pure Mathematics_ (London, 1886, pp. xxxvii, 935), and Hagen, _Synopsis der Höheren Mathematik_. The latter work by J. G. Hagen, Director of the Observatory, Georgetown College, Washington, D. C., is a stupendous undertaking, planned to be completed in four quarto volumes, of which two have appeared, and form most valuable works of reference.

8. But no one man can prepare the best possible compendium of all mathematics, or even of any large branch of it. Realizing this there is being prepared, under the auspices of the academies of science in Munich and Vienna, and
the Scientific Society of Göttingen, jointly, an *Encyclopedia of Mathematics* which is to be a compendium of the status of mathematical science today, with bibliographic references since the beginning of the nineteenth century. The work is planned in seven volumes, each in many parts, and over sixty prominent mathematicians are connected with the first three volumes, dealing with so-called pure mathematics (78 topics, each entrusted to a specialist). The work is now in process of appearing (first installment issued in 1898) and constitutes the best general survey of the field of mathematics which exists at present.*

9. Parallel with the encyclopedia, which is purely bibliographical, its publishers (Teubner, Leipzig) are also getting out a *series of mathematical textbooks*, which will include proofs as well as results, and which are intended to give a fuller survey of the present state of the various topics taken up than can be given within the narrow limits of the encyclopedia. The authors are mathematicians of high rank. In a number of instances the writer of a section in the encyclopedia furnishes a corresponding treatise as text-book, and the series will thus constitute a most valuable supplement to the encyclopedia.*

10. The *Jahresbericht der Deutschen Mathematiker Vereinigung*, founded 1891, publishes excellent, sometimes elaborate reports on the development and on the present status of special topics, which reports often constitute most valuable bibliographies. Thus there have already been published reports on the Theory of Invariants, the Theory of Algebraic Functions, the Theory of Algebraic Number-fields, Synthetic Geometry, text-books on Infinitesimal Calculus, Theory of Probabilities, Point Manifolds, Kinetic Problems of Scientific Technology, Development According to Oscillatory Functions, and a number of others are announced for the near future.†

11. There has just been published (1903) a German work, *Mathematischer Bücherschatz*, by Wölffing (Teubner, Leipzig), which aims to give a complete list of all non-periodical advanced mathematical works of the world, published in the Nineteenth century. The titles are arranged under 313 heads, and alphabetically by authors under these heads.

12. The *Bibliotheca Mathematica* (Stockholm), though primarily devoted to the history of mathematics, occasionally contains good bibliographies of special topics.

### IV. CLASSIFICATIONS OF MATHEMATICS.

Prerequisite to any general bibliographic work like any of those men-

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* A fuller account of the encyclopedia and the series will be given in a subsequent number of the *Monthly*.
† Meyer: *Bericht ueber d. gegenwassertigen Stand d. Invarianten-theorie*, 1890.
Hilbert: *Die Theorie d. algebraischen Zahlkoerper*, 1895.
E. Koetter: *Die Entwicklung d. synthetischen Geometrie*, 1897.
Bohmann: *Uebersicht ueber die wichtigsten Lehrbucher d. Infinitesimalrechnung von Euler bis auf die heutige Zeit*, 1897.
Czuber: *Die Entwicklung der Wahrscheinlichkeitslehre*, 1898.
Heun: *Die Kinetische Probleme d. wissenschaftlichen Technik*, 1900.
Burkhardt: *Entwicklungen nach oscillirnden Funktionen*, 1901.
tioned, is some classification of mathematical topics. The *Encyclopedia* entitles its seven volumes as follows (thus forming a first classification):

I. Arithmetic and Algebra
II. Analysis
III. Geometry
IV. Mechanics
V. Theoretic Physics
VI. Geodesy and Mathematical parts of Geophysics and Astronomy
VII. Philosophical, Historical, and Pedagogical Questions.

Each volume is subdivided into twenty or more topics which are assigned to various writers, each one of whom organizes the matter within his own topic as may seem best to him.

A somewhat similar classification is continued in the *Fortschritte*, while those of the *Reportoire* and of the Royal Society Catalogue are each more elaborate. The former has five indices (example, L'3b=1), while the latter is on a decimal system with four digits (example, 3470).

The simplicity of the latter system produces a favorable first impression, but as it has been made public only so recently it is not yet possible to speak concerning its merits in comparison with the earlier schemes of classification, or to prognosticate how well it will be found adapted to the actual work of classification and reference.

V. NECESSITIES OF MATHEMATICAL BIBLIOGRAPHY.

The bibliographical helps which are periodical (appearing annually or more frequently) cover the whole field in each issue. Topical bibliographies are also desirable with sub-classifications and reference to minor portions of works on other topics, which deal with the topic in hand.

The *Fortschritte* makes little attempt at sub-classification. The Royal Society Catalogue has a more elaborate classification and some cross references—but in a comprehensive bibliography of all mathematics sub-classification cannot with advantage be carried to the same extent as in the bibliography of a special topic.

Bibliographies of the latter type may achieve two important ends, viz:

1. All the material in a single classification independently of the year of publication.

2. Greater sub-classification than is possible in a more general bibliography.

Topical bibliographies, whether of large subjects or of narrow, highly special fields are now perhaps the most important bibliographic disiderata in mathematics. Much work of this sort has already been done. It would be a bibliographic undertaking of considerable magnitude and importance to prepare a fairly complete list of such special bibliographies.*

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*I mention simply a few by American writers:
At the International Congress of Mathematicians held at Paris in 1900, Mr. Ed. Maillet recommended the preparation and publication of bibliographic notices for the assistance of those who wish to take up the study of specific problems. He gives as a model:

"The last theorem of Fermat: \( x^m + y^m = z^m \), \( m > 2 \). Fermat announced without proof the theorem: The indeterminate equation \( x^m + y^m = z^m \) cannot be satisfied by integers if \( m > 2 \). This theorem has not yet been completely solved. To begin its study read:

- Serret: *Algèbre Supérieure*;
- Legendre: *Mém. de l' Institution*, 1823;
- Dedekind-Dirichlet: *Zahlentheorie*;
- Bachmann: *Kreistheilung*;
- (or, in place of the last two books, Bachmann: *Zahlentheorie*);
- Minimoff: *Journal für Mathematik*;

The first attack might be restricted to an attempt to prove that \( x^\lambda + y^\lambda = z^\lambda \), \( \lambda \) a prime, is impossible in prime integers, for every \( t \) superior to a certain limit function of \( \lambda \); the theorem is already established when \( x, y, z \) are prime to \( \lambda \), and among themselves.

Sketches like the above for a great number of mathematical problems, whether difficult or not, outlining in a few lines the state of the question, and the problems to be solved, are very desirable; *Intermédiaire* stands ready to publish them."

On the same general line, Hilbert read at the Congress a very important and inspiring paper on the future problems of mathematics. This paper may be found in French, in the *Congress Reports*, 1900; in German, in the *Gött. Nachrichten*, 1900; also in the *Archiv. f. Math. u. Physik*, 1900; in English, in the *Bulletin of the American Mathematical Society*, 1902.

What precedes relates to the newer topics and fields, those which are the scene of present mathematical activity, and where a good bibliography may contribute directly to further investigation.

Bibliographies in the field of the older and settled questions are, however, also desirable, both because these questions stand in more or less close relation to unsettled questions, and also because a careful study of the genesis of any notion, its treatment and development to a completed form is always instructive and stimulating.

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