Since it takes the policeman 1 minute to gain $\frac{1}{2}$ mile, it will take him 6 minutes to overtake the auto and end the race.

In the statement of this problem, the semi-colon should be omitted after the word "started." With this omission the problem was solved as above by J. W. Clawson, V. M. Spunar, J. H. Meyer, R. D. Carmichael, A. H. Holmes, B. Kramer, J. Scheffer, J. K. Ellwood, and P. S. Berg. Interpreting the problem as printed, agree in the answer being 2 hours, 36 minutes.

GEOMETRY.


A man owning a rectangular field $b=300$ feet by $a=600$ feet, wishes to lay out drive-ways of equal width having the diagonals of the field as center lines and such that the area of the driveways shall be $\frac{n}{m}=$one-half, of the area of the field. Determine the width of the driveways.

Solution by J. Scheffer, A. M., Hagerstown, Md.; Nellie Wood, Senior Class, Drury College, Springfield, Mo.; and A. H. Holmes, Brunswick, Me.

Put $AE=x$; then $AG=\frac{bx}{a}$; $\triangle EFK=\triangle GHL=\frac{b}{a} \cdot \frac{(a-2x)^2}{4}$.

\[ \therefore \text{space occupied by driveways} = ab - \frac{b}{a} (a-2x)^2. \]

\[ \therefore ab - \frac{b}{a}(a-2x)^2 = \frac{n}{m} ab; \text{ whence } \]

\[ x = \frac{a}{2} \left(1 - \sqrt{1 - \frac{n}{m}}\right). \]

\[ \therefore \text{breadth of driveway} = \frac{ab}{\sqrt{(a^2+b^2)}} \left(1 - \sqrt{1 - \frac{n}{m}}\right). \]

For $a=600$, $b=300$, $n:m=1:2$, we find breadth $=60(2\sqrt{5}-\sqrt{10})=78.572$ feet.


337. Proposed by T. N. Hildebrant, The University of Chicago.

Required the locus of the vertices of the parabolae having a given focus and passing through a given point.

Solution by the PROPOSER.

Let $O$ be the given focus and $P$ the given point. From the properties of the parabola we see that the directrices of the parabolae passing through $P$ will be the tangents to the circle of radius $OP$ and center $P$. Hence the vertices will be the mid-points $D$ of the perpendiculars from $O$ to the tangents. From elementary geometry we have the triangles $OCB$ and $OAB$ equal, and therefore $OC=OA=x_1$, the abscissa of $B$ the point of tangency, if we suppose $O$ the origin of our system of coordinates. Hence $OD=\frac{1}{2}x_1$. Denote by $a$ the angle $POC$. Then we also have $EPB=a$. Evidently
\[ x_1 = OP + PA = a + a \cos \alpha = a(1 + \cos \alpha); \text{ i.e., } OD = \frac{1}{2}x_1 = \frac{a}{2}(1 + \cos \alpha), \]

which gives as the polar equation of the locus referred to \( O \) as origin and \( OP \) as initial line,

\[ \rho = \frac{a}{2}(1 + \cos \alpha), \]

a cardioid, with cusp at \( O \) passing through \( P \) and having \( OP \) as line of symmetry.


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CALCULUS.

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The join of the center of curvature of a curve to the origin is at \( a \) to the initial line. Prove that with the usual notation:

\[ \frac{d \alpha}{d \phi} \left( \frac{d \rho}{d \phi} \right)^2 + \left( \frac{d^2 \rho}{d \phi^2} \right)^2 = \frac{d \rho}{d \phi} \frac{d \rho}{d \phi} \]


Denote the coordinates of the center of curvature by \( \xi, \eta \). Then \( \xi = x - \rho \sin \phi, \eta = y + \rho \cos \phi \), \( \rho \) being the radius of curvature and \( \phi \) the angle formed by the tangent and positive \( x \)-axis.

\[ \tan a = \eta/\xi, \text{ and } \frac{d \alpha}{d \phi} = \frac{\xi \frac{d \eta}{d \phi} - \eta \frac{d \xi}{d \phi}}{\xi^2 + \eta^2}. \]

Likewise, \( \tan \psi = \frac{d \theta}{d \rho} \), and \( d \psi \left[ d \theta + r \left( \frac{d^2 \theta}{d \rho^2} \right) \right] + \sec^2 \psi \), where \( \psi \) = angle included by the tangent at \( (r, \theta) \), and the radius vector to the point \( (r, \theta) \).

Substituting the proper values of \( \xi, \eta, d \xi, d \eta \) in \( d \alpha \), expressed in polar coordinates, and we have the first product \( d \alpha/d \phi \).

Also, on remembering that \( \begin{cases} p = r \sin \psi \\ \rho = \theta + \psi \end{cases} \), differentiate it twice, square every differential quotient as indicated by the proposition, add the like terms, reduce, and we have, readily,

\[ \frac{d \alpha}{d \phi} \left( \frac{d \rho}{d \phi} \right)^2 + \left( \frac{d^2 \rho}{d \phi^2} \right)^2 = \frac{d \rho}{d \phi} \frac{d \rho}{d \phi} \]


Find two curves which possess the property that the tangents \( TP \) and \( TQ \) to the inner one always makes equal angles with the tangent \( TT' \) to the outer.